Indirect Model Reference Adaptive Control System Based on Dynamic Certainty Equivalence Principle and Recursive Identifier Scheme

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Abstract

The direct scheme of Model Reference Adaptive Control System (MRACS) may have several disadvantages; (i) the current state of the plant cannot be easily grasped from values of adjustable adaptive controller parameters, (ii) it has often inferior conditions to an indirect scheme, with respect to the persistent excitation (PE) conditions of adjustable controller parameters for convergence to optimal values. Indirect MRACS based on Dynamic Certainty Equivalent (DyCE) principle hasnot been well studied though the importance of indirect scheme is recognized in an actual field using adaptive control. This paper presents new two design schemes of indirect MRACS based on DyCE principle. These are realized by recursive identifiers and new high order tuners.

Keywords

Model Reference Adaptive Control System; Indirect Scheme; Dynamic Certainty Equivalence Principle; Recursive Identifier

Introduction

The design method of Model Reference Adaptive Control System (MRACS) based on the Surrogate Model Control (SMC) scheme is originally proposed by Morse (Morse, 1992). The SMC obtained by a control strategy that the output of a certain identifier is always equivalent to the desired value as an output of reference model of control system, has several advantages as follows. It has the ability to apply MRAC to a certain plant; for example, a plant with nonlinearities which do not satisfy globally Lipchitz conditions, a LTI plant whose relative degree is n^*or n^*+1, n^* being a known positive integer and so on. Additionally, MRACS based on SMC scheme is superior in a transient response of controlled variable to the conventional MRACS based on certainty equivalence (CE) principle since the SMC input includes an auxiliary input which prevents the derivative of adjustable controller parameter from appearing in the tracking error. Hence, the swapping term in the tracking error in MRACS with CE control is perfectly removed by the auxiliary input. The adaptive control input by SMC scheme can be expressed as a result in a form that the auxiliary input is added to the CE adaptive control input. Therefore, it is also called an adaptive control based on the dynamic certainty equivalence (DyCE) principle. However, it needs high order differential values of adjustable parameters which must be updated by an adaptation law. Morse (Morse, 1992) and Ortega (Ortega, 1993) proposed a series of the high order tuner (HOT) being special adaptation laws, which can generate high order differential values of adjustable controller parameters though it does not use high order differential values of the control variable.

Several modified schemes of MRACS due to DyCE principle have been proposed [for example, (Itamiya, 1998), (Masuda, 2000), (Tanahashi, 2007), (Tanahashi, 2011)] in consideration of the realization of HOT, some improvement of control performance and so on. However, the scheme based on SMC in which estimates of plant parameters are used directly for adaptive controller hasnot been well studied. In conventional MRACS based on CE control, such a scheme is well known as indirect one. Advantages in indirect method are (i) one can observe or predict a behaviour of plant through estimates, (ii) fast convergence property of estimates and small

parameter error may be expected since PE degree for an adaptation law is less than that in a direct MRACS. Especially, the advantage (i) can is never obtained in any direct MRACS but it is significant from practical viewpoints.

Two new schemes of MRACS with DyCE control have been proposed here using estimates of plant parameters directly. These can be constructed by introducing certain *recursive identifiers* and HOTs induced as steepest descent methods which minimize certain integral costs. These HOTs are similar to the adaptive law proposed by Kreisselmeier (Kreisselmeier, 1977). In the schemes proposed here, any design calculation with nonlinear algebra isnot needed. The controllers are constructed directly by estimates of plant parameters.

This paper is organized as follows; the next section states the system representation and the problem statement. The third Section presents the scheme 1 to design a MRACS with DyCE control directly using estimates of plant parameters and its brief stability analysis. The scheme 2 such that the steady state response of control variable is improved and its stability analysis is shown in the fourth section. Simple numerical simulation results in order to verify the effectiveness of the proposed schemes are indicated in the fifth section. The last section concludes the paper.

Notations: The following notations are applied; s denotes Laplace operator. The symbol $^{\mathrm{T}}$expresses the transpose of a vector or a matrix. A definition symbol is expressed as: =and $\|\cdot\|$ means Euqulidean norm or 2 norm of Matrix induced from Euclidean norm. The symbol L_2 and L_∞ mean the set of square integrable functions and uniformly bounded functions, respectively. Furthermore,

$$(G(s)[v])(t) := \int_0^t g(t-\tau)\, v(\tau)\, \mathrm{d}\tau$$

$$x^{[j]}(t) := \begin{cases} ((s+\lambda)^j[x])(t) \text{ for } j \le 0 \\ (D+\lambda)^j x(t) \text{ for } j > 0 \end{cases}$$

where $G(s)$ is the Laplace transforms of $g(t)$, $j$means integer, λ is a positive constant and D is a time differentialoperator $\frac{d}{dt}$ and $x^{(j)} \equiv \frac{\mathrm{d}^j}{\mathrm{d}t^j}x(t)$ norm is defined as

$$\| x_t \|_{2\delta} := \left(\int_0^t e^{-\delta(t-\tau)} \| x(\tau) \|^2 \, \mathrm{d}\tau \right)^{1/2}$$

for $x(\cdot) \in L_{2e}$ and $\delta > 0$. L_{2e} means the set of square integrable function in finite interval. $\| W(s) \|_\infty$ is the H_∞norm of a proper and stable transfer matrix $W(s)$. In addition, $\| W(s) \|_2$ is the H_2 norm of a strictly proper and stable transfer matrix $W(s)$.

System Representation and Problem Statement

The control object (plant) considered here is a LTI, SISO and minimum phase system. It has a coprime transfer function $N_{\mathrm{P}}(s)/D_{\mathrm{P}}(s)$ where degrees of polynomials $D_{\mathrm{P}}(s)$ and $N_{\mathrm{P}}(s)$ are n and $n-n^*$ respectively. When $u(t)$ is the plant input in time t, the plant output $y(t)$ can be represented as

$$y(t) = \theta^{\mathrm{T}}\zeta(t) + \epsilon(t) \quad (1)$$

where $\epsilon(t)$ is a exponentially decaying term whose size depends on the initial value of the plant initial state, the plant parameter $\theta \in \mathbb{R}^{2n-n^*+1}$ and $\zeta(t) \in \mathbb{R}^{2n-n^*+1}$ are defined by

$$\left.\begin{aligned} \theta &:= [\theta_u^{\mathrm{T}}, \bar{\theta}_y^{\mathrm{T}}, \theta_y^{\mathrm{T}}]^{\mathrm{T}} \\ \theta_u &:= [\theta_{n^*}, \theta_{n^*+1}, \dots, \theta_n]^{\mathrm{T}} \\ \bar{\theta}_y &:= [\theta_{n+1}, \theta_{n+2}, \dots, \theta_{n+n^*-1}]^{\mathrm{T}} \\ \theta_y &:= [\theta_{n+n^*}, \theta_{n+n^*+1}, \dots, \theta_{2n}]^{\mathrm{T}} \end{aligned}\right\} (2)$$

$$\left.\begin{aligned} \zeta(t) &:= [\zeta_u^{\mathrm{T}}(t), \bar{\zeta}_y^{\mathrm{T}}(t), \zeta_y^{\mathrm{T}}(t)]^{\mathrm{T}} \\ \zeta_u(t) &:= [u^{[-n^*]}(t), u^{[-(n^*+1)]}(t), \dots, u^{[-n]}(t)]^{\mathrm{T}} \\ \bar{\zeta}_y &:= [y^{[-1]}(t), y^{[-2]}(t), \dots, y^{[-(n^*-1)]}(t)]^{\mathrm{T}} \\ \zeta_y &:= [y^{[-n^*]}(t), y^{[-(n^*+1)]}(t), \dots, y^{[-n]}(t)]^{\mathrm{T}} \end{aligned}\right\} (3)$$

Remark 1: For any $\lambda > 0$, $D_{\mathrm{p}}(s)$ and $N_{\mathrm{p}}(s)$ can be generally represented as

$$D_{\mathrm{p}}(s) = \sum_{i=0}^{n} a_i\, (s+\lambda)^{n-i}\,;\ a_0 = 1 (4)$$

$$N_{\mathrm{P}}(s) = \sum_{i=n^*}^{n} b_i\, (s+\lambda)^{n-i} (5)$$

Then, the next relationship holds;

$$\begin{aligned} \theta_i &= b_i \quad && (i = n^* \sim n) \\ \theta_{n+i} &= -a_i \quad && (i = 1 \sim n) \end{aligned} (6)$$

The followings are assumed for the plant;

Assumption 1:

A1) The system degree n and the relative degree n^* are known *a priori.*

A2) θ is unknown.

A3) The sign of θ_{n^*} is known *a priori.* Here, $\theta_{n^*} > 0$ without loss of generality.

A4) Available signals are only $u(t)$ and $y(t)$.

The problem to be considered is to design the MRACS with DyCE control which uses estimates of plant parameters. The desired output that the plant output

$y(t)$ must follow is given by the reference model output $y_{\mathrm{m}}(t)$;

$$y_{\mathrm{m}}(t) := \left(\frac{N_{\mathrm{m}}(s)}{D_{\mathrm{m}}(s)}[r]\right)(t) \quad (7)$$

where $r(t)$ is a piece-wise continuous and bounded reference input, $D_{\mathrm{m}}(s)$ is a monic and Hurwitz polynomial with degree n_{m}, $N_{\mathrm{m}}(s)$ is a polynomial whose degree is less than or equal to $n_{\mathrm{m}} - n^*$. The transfer function is the desired reference model for the closed loop system.

Scheme 1

Design of MRACS

Now, consider the following linear *recursive identifier* corresponding to (1);

$$\left.\begin{aligned} \hat{y}_{11}(t) &:= \hat{\theta}^{\mathrm{T}}(t)\zeta(t) \\ \hat{y}_{1j}(t) &:= \hat{\theta}^{\mathrm{T}}(t)\psi_j(t) \quad (j = 2 \sim n^* - 1) \\ \hat{y}_{1n^*}(t) &:= \hat{\theta}^{\mathrm{T}}(t)\psi_{n^*}(t) \end{aligned}\right\} \quad (8)$$

where $\hat{\theta}(t) \in \mathbb{R}^{2n-n^*+1}$ is the estimate of θ, $\psi_j(t)$ $(j = 2 \sim n^*)$ is defined as follows;

$$\left.\begin{aligned} \psi_j(t) &:= [\zeta_u^{\mathrm{T}}(t), \bar{\zeta}_{\hat{y}j}^{\mathrm{T}}(t), \bar{\zeta}_{yj}^{\mathrm{T}}(t), \zeta_y^{\mathrm{T}}(t)]^{\mathrm{T}} \\ \bar{\zeta}_{\hat{y}j}(t) &:= [\hat{y}^{[-1]}(t), \hat{y}^{[-2]}(t), \dots, \hat{y}^{[-(j-1)]}(t)]^{\mathrm{T}} \\ \bar{\zeta}_{yj}(t) &:= [\hat{y}^{[-j]}(t), \hat{y}^{[-(j+1)]}(t), \dots, \hat{y}^{[-(n^*-1)]}(t)]^{\mathrm{T}} \end{aligned}\right\} \quad (9)$$

$$\left.\begin{aligned} \psi_{n^*}(t) &:= [\zeta_u^{\mathrm{T}}(t), \bar{\zeta}_{\hat{y}n^*}^{\mathrm{T}}(t), \zeta_y^{\mathrm{T}}(t)]^{\mathrm{T}} \\ \zeta_{\hat{y}n^*}(t) &:= [\hat{y}_{1n^*-1}^{[-1]}, \hat{y}_{1n^*-2}^{[-2]}, \dots, \hat{y}_{11}^{[-(n^*-1)]}]^{\mathrm{T}} \end{aligned}\right\} \quad (10)$$

Since $\hat{y}_{1j}(t)$ is j times differentiable on t if $\hat{\theta}^{(k)}(t)$ $(k = 0 \sim j)$ is available, here an adaptive control input can be proposed so as to satisfy the SMC law;

$$\hat{y}_{1n^*}(t) = y_{\mathrm{m}}(t) \quad (11)$$

The control law (11) leads to

$$\tilde{y}(t) = y(t) - y_{\mathrm{m}}(t) = \sum_{j=1}^{n^*} \varepsilon_{1j}(t) \quad (12)$$

where $\varepsilon_{1j}(t)$ is defined as

$$\left.\begin{aligned} \varepsilon_{11}(t) &:= y(t) - \hat{y}_{11}(t) \\ \varepsilon_{1j}(t) &:= \hat{y}_{1j-1}(t) - \hat{y}_{1j}(t) \quad (j = 2 \sim n^*) \end{aligned}\right\} \quad (13)$$

The equation (11) means

$$u(t) = \frac{r'(t) - \bar{\hat{\theta}}^{T}(t)\bar{\psi}_{n^*}^{[n^*]}(t) - f_1(t)}{\hat{\theta}_{n^*}(t)} \quad (14)$$

where $\bar{\hat{\theta}}(t) \in \mathbb{R}^{2n-n^*}$ and $\bar{\psi}(t) \in \mathbb{R}^{2n-n^*}$ are sub-vectors which consist of elements exception of the first component in $\hat{\theta}(t)$ and $\psi_{n^*}(t)$ respectively. $r'(t)$ and $f_1(t)$ are defined as

$$\left.\begin{aligned} r'(t) &:= \left(\frac{(s+\lambda)^{n^*} N_m(s)}{D_m(s)}[r]\right)(t) \\ f_1(t) &:= \sum_{j=1}^{n^*} {}_{n^*}\mathrm{C}_j \cdot \left(\hat{\theta}^{(j)}(t)\right)^{\mathrm{T}} \psi_{n^*}^{[n^*-j]}(t) \end{aligned}\right\} \quad (15)$$

Remark 2: It can be shown that (14) is equivalent to the adaptive law used in a usual indirect MRACS if $f_1(t) \equiv 0$.

Remark 3: In a direct scheme MRACS with SMC, the control input can be realized because the regressor vector is n^* times differentiable. However, the surrogate model control

$$\hat{y}_{11}(t) = y_{\mathrm{m}}(t) \quad (16)$$

cannot be realized since $\zeta^{(j)}(t)$ $(j = 2 \sim n^*)$ cannot be obtained without using $\dot{y}(t) \sim y^{(n^*-1)}(t)$.

The control law (14) needs the estimate $\hat{\theta}(t)$ of plant parameter and its high order differential values. Therefore, we aim at the following relationship;

$$\left.\begin{aligned} \varepsilon_{11}(t) &= \tilde{\theta}^{\mathrm{T}}(t)\,\zeta(t) + \epsilon(t) \\ \varepsilon_{1j}(t) &= \sum_{i=1}^{j-1} \hat{\theta}_{n+i}(t)\,\varepsilon_{1j-i}^{[-i]}(t) \end{aligned}\right\} \quad (17)$$

where $\tilde{\theta}(t) := \theta - \hat{\theta}(t)$ and $j = 2 \sim n^*$. From (17) and (12), it is clear that $\varepsilon_{ij}(t)$ for $j = 2 \sim n^*$ converges to zero and then also $\tilde{y}(t)$ converges to zero if $\hat{\theta}_{n+i}(\cdot) \in L_\infty$ for $i = 1 \sim n^* - 1$ and $\varepsilon_{11}(t)$ vanishes. The following HOT can guarantee such a property;

[HOT1]

$$\dot{\hat{\theta}}(t) = \Gamma_1(t)\{q_1(t) - R_1(t)\} \quad (18)$$

where

$$\Gamma_1(t) := \Gamma_{10}\, P_1(t) \quad (19)$$

$$\Gamma_{10} := \mathrm{diag}[\gamma_{1n^*}, \mathrm{diag}\{\gamma_{1n^*+1}, \gamma_{1n^*+2}, \dots, \gamma_{12n}\}] \quad (20)$$

$$P(t) := \mathrm{diag}\{\hat{\theta}_{n^*}(t), I_{2n-n^*}\}/N_{10}(t) \quad (21)$$

$$N_{10}(t) := \sqrt{\hat{\theta}_{n^*}^2(t) + 2n - n^*} \quad (22)$$

$$q_1(t) := \left(\frac{\lambda_1^{n^*}}{(s+\lambda_1)^{n^*}}[y_{\mathrm{N}}\,\zeta_{\mathrm{N}}]\right)(t) \quad (23)$$

$$R_1(t) := \left(\frac{\lambda_1^{n^*}}{(s+\lambda_1)^{n^*}}[\zeta_{\mathrm{N}}\,\zeta_{\mathrm{N}}^{\mathrm{T}}]\right)(t) \quad (24)$$

where $\lambda_1 > 0$ and $\gamma_{1j} > 0$ $(j = n^* \sim 2n)$. $y_{\mathrm{N}}(t) := y(t)/N_1(t)$ and $\zeta_{\mathrm{N}}(t) := \zeta(t)/N_1(t)$ mean normalized signals where $N_1(t)$ is defined as follows;

$$N_1(t) := \sqrt{\rho_1 + \zeta^{\mathrm{T}}(t)\zeta(t)}\,;\ \rho_1 > 0 \quad (25)$$

Initial estimates are given as $0 < \hat{\theta}_{n^*}(0) < \infty$ and $\| \bar{\hat{\theta}}(0) \| < \infty$. Note that from (18)–(24), high order

differential values of $\hat{\theta}(t)$ can be obtained without using high order differential values of $y(t)$.

Remark4: It can be seen that

$$q_1(t)-R_1(t)\,\hat{\theta}(t)=\frac{1}{2}\left(-\frac{\partial J_1(t)}{\partial\,\hat{\theta}(t)}\right)^{\mathrm{T}}(26)$$

$$J_1(t):=\int_0^t h_1(t-\tau)\{y_{\mathrm{N}}(\tau)-\hat{\theta}^{\mathrm{T}}(t)\,\zeta_{\mathrm{N}}(\tau)\}^2\mathrm{d}\tau(27)$$

where $h_1(t)$ means the inverse Laplace transform of $\lambda_1^{n^*}/(s+\lambda_1)^{n^*}$.

Remark 5: The term, $\{q_1(t)-R_1(t)\,\hat{\theta}(t)\}^{\mathrm{T}}$, is the steepest descent direction of $J_1(t)$ for $\hat{\theta}(t)$. It is resemble to one in Kreisselmeier's adaptive law (Kreisselmeier, 1977). The definition of $h(t)>0$ guarantees the existence of high order differential values of $\hat{\theta}(t)$.

Furthermore, $\hat{\theta}_{n^*}(t)$ in the definition (19) of $\Gamma(t)$ contributes to the fact $\hat{\theta}_{n^*}(t)>0$ for any non-negative t (Sawada, 2013). Hence, conventional (switching) projection algorithm[(Goodwin,1987), (Ikhouane,1998), (Nagurney, 1996), (Kuhnen, 2004)] is not needed to satisfy $\hat{\theta}_{n^*}(t)>0$.

The normalizing signals $N_{10}(t)$ and $N_1(t)$ establish $\| P_1(t)\| < 1$ and $|y_{\mathrm{N}}(t)| < \|\theta\| + c$, $\|\zeta_{\mathrm{N}}(t)\| < 1$. These results contribute to $P_1(\cdot)$, $q_1(\cdot)$, $R_1(\cdot)\in L_\infty$. Thus, $\dot{\hat{\theta}}(\cdot)\in L_\infty$ and the upper bound of $\|\dot{\hat{\theta}}(t)\|$ can be designed by Γ_{10} if $\theta-\hat{\theta}(\cdot)\in L_\infty$.

After all, the scheme1 proposed here consists of the control law (14) and the HOT (18).

Stability of MRACS Based on Scheme 1

Firstly, the stability of the adaptive loop has been shown by scheme1. The next lemma holds.

Lemma 1 *(Stability of adaptive loop):* The HOT (18) of scheme 1 satisfies the following properties;

P1-1) $\hat{\theta}^{(j)}(\cdot)\in L_\infty$.
P1-2) $J_1(\cdot)\in L_1$.
P1-3) $\dot{\hat{\theta}}(\cdot)\in L_2$.
P1-4) $\hat{\theta}_{n^*}(t)>0$ for all $t\geq 0$.
P1-5) $\varepsilon_{11\mathrm{N}}(\cdot)\in L_2$
where $\varepsilon_{11\mathrm{N}}(t):=\varepsilon_{11}(t)/N_1(t)$.

proof: Let the positive definite function $V_1(\tilde{\theta}(t))$ be

$$V_1(\tilde{\theta}(t)):=\frac{1}{\gamma_{1n^*}}\left\{-\tilde{\theta}_{n^*}(t)+\theta_{n^*}\ln\frac{\theta_{n^*}}{\hat{\theta}_{n^*}(t)}\right\}+\sum_{i=n^*+1}^{2n}\frac{\tilde{\theta}_i^2(t)}{2\gamma_{1i}};$$
$$\tilde{\theta}_i:=\theta_i-\hat{\theta}_i(t)(28)$$

which is defined in the set;

$$C:=\left\{\hat{\theta}(t)\in\mathbb{R}^{2n-n^*+1}|\hat{\theta}_{n^*}(t)>0\right\}(29)$$

Then, the time differentiation of $V_1(\tilde{\theta}(t))$ evaluated along the trajectory of (18) becomes

$$\dot{V}_1(\tilde{\theta}(t))=-J_1(t)/N_{10}(t)\leq 0(30)$$

Therefore, $V_1(\tilde{\theta}(\cdot))$, $\hat{\theta}(\cdot)\in L_\infty$ and $V_1(\tilde{\theta}(t))$ converges some constant in $[0, V_1(\tilde{\theta}(0))]$. Hence, P1-1) is proven because $q_1^{(j)}(\cdot)$, $R_1^{(j)}(\cdot)\in L_\infty$ for $j=0\sim n^*$. From the integration of (30), P1-2) is proven. P1-3) holds since $\|\dot{\hat{\theta}}(t)\|^2$ can be evaluated as

$$\|\dot{\hat{\theta}}(t)\|^2\leq\|\Gamma_1\|^2\{\mathrm{trace}\,R_1(t)\}^2 J_1(t)(31)$$

and $J_1(\cdot)\in L_1$. P1-4) is shown by the fact

$$\lim_{\hat{\theta}_{n^*}(t)\to 0} n_{c0}^{\mathrm{T}}\,\Gamma_1(t)\{q_1(t)-R_1(t)\,\hat{\theta}(t)\}=0(32)$$

where $n_{c0}:=[-1,\,0^{\mathrm{T}}]^{\mathrm{T}}$ is the normal vector at $\hat{\theta}_{n^*}(t)=0$ of the convex set C. Also, P1-5) is proven by P1-2) and the mathematical induction similar to the paper (Tanahashi, 2011). Q. E. D.

From the above lemma 1, the following theorem about the main control loop (closed loop) holds.

Theorem 1 *(Stability):* All variables in the adaptive control system which consists of (1), (14) and (18) are uniformly bounded in t, and

$$\lim_{t\to\infty}\tilde{y}(t)=0(33)$$

proof: In the following, the exponential decaying term is abbreviated because it does not affect the stability. Note that $\zeta(t)$ can be represented as

$$\left.\begin{aligned}&\zeta(t)=(z(s)[y])(t)\\&z(s):=[\tfrac{D_p(s)}{(s+\lambda)^{n^*}},\tfrac{D_p(s)}{(s+\lambda)^{n^*+1}},\ldots,\tfrac{D_p(s)}{(s+\lambda)^{n}},\\&\tfrac{1}{s+\lambda},\tfrac{1}{(s+\lambda)^2},\ldots,\tfrac{1}{(s+\lambda)^n}]^{\mathrm{T}}\end{aligned}\right\}(34)$$

which satisfies (12) and $z(s)$ can be decomposed into

$$\left.\begin{aligned}&z(s)=b+\bar{z}(s)\\&b:=[1/\theta_{n^*},\,0_{2n-n^*}]^{\mathrm{T}}\end{aligned}\right\}(35)$$

where $\bar{z}(s)$ is a stable and strictly proper transfer vector.

Then, the following relationship can be obtained.

$$M(t)\,\zeta(t)=(z(s)[y_{\mathrm{m}}])(t)+b\sum_{j=2}^{n^*}\varepsilon_{1j}(t)$$

$$+\left(\bar{z}(s)\left[\sum_{j=1}^{n^*}\varepsilon_{1j}\right]\right)(t)(36)$$

where $M(t)$ is the upper triangular matrix as

$$M(t):=\left[\frac{\beta^{\mathrm{T}}(t)}{0_{2n-n^*}I_{2n-n^*}}\right](37)$$

$$\beta(t):=[\hat{\theta}_{n^*}(t),\ -\tilde{\bar{\theta}}^{\mathrm{T}}(t)]^{\mathrm{T}}/\theta_{n^*}(38)$$

which is clearly non-singular from P1-4) of the lemma 1. Therefore,

$$\| \zeta(t) \|^2 \leq c + c\sum_{j=2}^{n^*} | \varepsilon_{1j}(t)|^2 + c\sum_{j=1}^{n^*} \| \varepsilon_{1jt} \|_{2\delta}^2 (39)$$

where $c > 0$ is a generic constant. In the derivation of above inequality, the next relationships are used;

$$\left|\sum_{i=1}^{k} x_i\,(t)\right|^2 \leq k\sum_{i=1}^{k} | x_1(t)|^2 (40)$$

$$\| (\bar{z}(s)[\varepsilon_{1j}])(t) \|^2 \leq \| \bar{z}(s) \|_{2\delta}^2 \cdot \| \varepsilon_{1jt} \|_{2\delta}^2 (41)$$

where $\| \bar{z}(s) \|_{2\delta}$ means the H_2 norm of $\bar{z}(s-\delta/2)$. δ satisfies $0<\delta<\delta_0$; $\delta_0:=\min\{2\lambda,\ 2\lambda_{\mathrm{Np}}\}$. λ_{Np} means $\min_i\{|\mathrm{Re.}(s_{\mathrm{Np}i})|\}$ where $s_{\mathrm{Np}i}$ $(i=1\sim n^*)$ is a zero of $N_{\mathrm{p}}(s)$.

Moreover, from (17) and $\hat{\theta}(\cdot)\in L_\infty$, $\varepsilon_{1j}(t)$ $(j=2\sim n^*)$ is evaluated as

$$\left.\begin{array}{l} |\varepsilon_{1j}(t)|^2 \leq c\ \| \varepsilon_{1j-1t} \|_{2\delta}^2 \\ \| \varepsilon_{1jt} \|_{2\delta}^2 \leq c\ \| \varepsilon_{1j-1t} \|_{2\delta}^2 \end{array}\right\} (42)$$

where $c>0$ is a generic constant, by using the concept of $H_{2\delta}$ norm and $H_{\infty\delta}$ norm (Ioannou, 1996). Therefore, there exists in some generic constant $c>0$ such that

$$\left.\begin{array}{l} |\varepsilon_{1j}(t)|^2 \leq c\ \| \varepsilon_{11t} \|_{2\delta}^2 \\ \| \varepsilon_{1jt} \|_{2\delta}^2 \leq c\ \| \varepsilon_{11t} \|_{2\delta}^2 \end{array}\right\} (43)$$

Inequalities (39), (43), the definition of $N_1(t)$ and the property P1-5) lead to

$$\| \zeta(t) \|^2 \leq c + c\int_0^t \varepsilon_{11\mathrm{N}}^2(\tau) \| \zeta(\tau) \|^2\ \mathrm{d}\tau(44)$$

This is equivalent to

$$\| \zeta(t) \|^2 \leq c\ \exp\left\{c\int_0^t \varepsilon_{11\mathrm{N}}^2(\tau)\,\mathrm{d}\tau\right\}(45)$$

from Bellman-Gronwall lemma (Ioannou, 1996). This means $\zeta(\cdot)\in L_\infty$ from P1-5). Hence, $y(\cdot)\in L_\infty$ from (1). $u(\cdot)\in L_\infty$ is guaranteed from *linear boundedness condition* (Narendra, 1989) or *regularity* (Sastry, 1989) since the controlled object is a minimum phase system, $y(\cdot)\in L_\infty$ and the fact that $u(t)$ grows at most exponentially from $\hat{\theta}(\cdot)\in L_\infty$.

The result of $u(\cdot), y(\cdot)\in L_\infty$ means $\varepsilon_{11}(\cdot)\in L_\infty\cap L_2$ and $\dot{\zeta}(\cdot)\in L_\infty$. So that, $\varepsilon_{11}(t)$ is uniformly continuous signal from $\hat{\theta}(\cdot), \dot{\hat{\theta}}(\cdot), \zeta(\cdot), \dot{\zeta}(\cdot)\in L_\infty$ and

$$\dot{\varepsilon}_{11}(t)=\dot{\hat{\theta}}^{\mathrm{T}}(t)\zeta(t)+\tilde{\theta}^{\mathrm{T}}(t)\dot{\zeta}(t)(46)$$

These facts lead to zero convergence of $\varepsilon_{11}(t)$ by Barbălat's lemma. Therefore, $\tilde{y}(t)$ converges to zero from (17) and (12). Q. E. D.

Scheme 2

The scheme 1 proposed in the previous section is able to achieve the property that MRACS is stable and the tracking error $\tilde{y}(t)$ onverges to zero. However, $\tilde{y}(t)$ and $\varepsilon_{11}(t)$ can never go to zero simultaneously. Meanwhile, $\tilde{y}(t)$ may have ill convergence property if $\hat{\theta}_{n+i}(t)$ for $i=1\sim n^*-1$ remain as large values since the tracking error equation is described by (12) with (17). In order to reveal such a phenomenon, another scheme is proposed here.

Design of MRACS

Consider another recursive identifiers corresponding to (1) as

$$\left.\begin{array}{l} \hat{y}_{21}(t):=\hat{\theta}^{\mathrm{T}}(t)\,\zeta(t) \\ \hat{y}_{2j}(t)=\hat{y}_{2j-1}(t) \\ \qquad -\alpha_{j-1}(t)\{y^{[-(j-1)]}(t)-\hat{\theta}^{\mathrm{T}}(t)\,\zeta^{[-(j-1)]}(t)\} \\ \alpha_{j-1}(t):=\sum_{i=1}^{j-1}\hat{\theta}_{n+i}(t)\,\alpha_{j-1-i}(t);\ \alpha_0(t)=1 \\ \qquad\qquad (j=2\sim n^*) \end{array}\right\}(47)$$

Then, $\hat{y}_{2j}(t)$ in (47) is j times continuously differentiable without the derivatives of $y(t)$. For example, when $n=n^*=2$, $\hat{y}_{22}(t)$ is evaluated from (47) as follows;

$$\begin{array}{l}\hat{y}_{22}(t)=\hat{\theta}_2(t)\,u^{[-2]}(t) \\ \qquad +\hat{\theta}_3(t)\,\hat{\theta}^{\mathrm{T}}(t)\,\zeta^{[-1]}(t)+\hat{\theta}_4(t)y^{[-2]}(t)(48)\end{array}$$

It is clearly 2 times continuously differentiable without derivatives of $y(t)$ if $\hat{\theta}(t)$, $\dot{\hat{\theta}}(t)$ and $\ddot{\hat{\theta}}(t)$ are available.

As the adaptive control law in scheme, the next SMC law has been put forward;

$$\hat{y}_{2n^*}(t)=y_{\mathrm{m}}(t)(49)$$

The control input is obtained by solving the following equation on $u(t)$;

$$\hat{y}_{2n^*}^{[n^*]}(t)=y_{\mathrm{m}}^{[n^*]}(t)(50)$$

According to (49), the tracking error $\tilde{y}(t):=y(t)-y_{\mathrm{m}}(t)$ becomes

$\tilde{y}(t) = \bar{y}(t) - \hat{\theta}^{\mathrm{T}}(t)\,\varphi(t)$(51)

$= \tilde{\theta}^{\mathrm{T}}(t)\,\varphi(t) + \bar{\epsilon}(t)$(52)

where

$\bar{y}(t) := \sum_{i=0}^{n^*-1} \alpha_i\,(t) y^{[-i]}(t)$(53)

$\varphi(t) := \sum_{i=0}^{n^*-1} \alpha_i\,(t)\,\zeta^{[-i]}(t)$(54)

$\bar{\epsilon}(t) := \sum_{i=0}^{n^*-1} \alpha_i\,(t) \epsilon^{[-i]}(t)$(55)

Since $\tilde{y}(t)$ in (52) is affine on $\tilde{\theta}(t)$, and $\bar{\epsilon}(t)$ decays exponentially if $\hat{\theta}_{n+i}(\cdot)$ $(i = 1 \sim n^* - 1) \in L_\infty$, it means that $\tilde{y}(t)$ goes to zero if $\tilde{\theta}^{\mathrm{T}}(t)\,\varphi(t) + \bar{\epsilon}(t)$ tends to zero as t increases. Therefore, $\hat{\theta}(t)$ and its high order differential values can be updated by

[HOT 2]

$\dot{\hat{\theta}}(t) = \Gamma_2(t)\{q_2(t) - R_2(t)\,\hat{\theta}(t)\}$(56)

where

$\Gamma_2(t) := \Gamma_{20}\,P_2(t)$(57)

$\Gamma_{20} := \mathrm{diag}[\gamma_{2n^*}, \mathrm{diag}\{\gamma_{2n^*+1}, \gamma_{2n^*+2}, \ldots, \gamma_{22n}\}]$(58)

$P_2(t) := \mathrm{diag}\{\hat{\theta}_{n^*}(t), I_{2n-n^*}\}/N_{20}(t)$(59)

$N_{20}(t) := \sqrt{\hat{\theta}_{n^*}^2(t) + 2n - n^*}$(60)

$q_2(t) := \left(\frac{\lambda_2^{n^*}}{(s+\lambda_2)^{n^*}}\,[\bar{y}_{\mathrm{N}}\,\varphi_{\mathrm{N}}]\right)(t)$(61)

$R_2(t) := \left(\frac{\lambda_2^{n^*}}{(s+\lambda_2)^{n^*}}\,[\varphi_{\mathrm{N}}\,\varphi_{\mathrm{N}}^{\mathrm{T}}]\right)(t)$(62)

where $\lambda_2 > 0$ and $\gamma_{2j} > 0$ $(j = n^* \sim 2n)$ are adaptive gains. $\varphi_{\mathrm{N}}(t) := \varphi(t)/N_2(t)$ means normalized signals where $N_2(t)$ is defined as follows;

$N_2(t) := \sqrt{\rho_2 + \varphi^{\mathrm{T}}(t)\varphi(t)}\,;\ \rho_2 > 0$(63)

Initial estimates are given as $0 < \hat{\theta}_{n^*}(0) < \infty$, $\|\,\hat{\theta}(0)\,\| < \infty$.

Note that from (56)–(63), high order differential values of $\hat{\theta}(t)$ can be obtained without using high order differential values of $y(t)$.

Clearly, high order differential values of $\hat{\theta}(t)$ can be obtained from (56) without derivatives of $y(t)$.

Remark 6: (56) is induced from the steepest descent method with smooth projection to minimize the following cost function;

$J_2(t) = \int_0^t h_2(t-\tau)\{\bar{y}_{\mathrm{N}}(\tau) - \hat{\theta}^{\mathrm{T}}(t)\,\varphi_{\mathrm{N}}(\tau)\}^2\,\mathrm{d}\tau$(64)

where $h_2(t)$ means the inverse Laplace transform of $\lambda_2^{n^*}/(s+\lambda_2)^{n^*}$.

The total surrogate model controller using plant parameters estimates based on the scheme 2 consists of the control law (49) and the HOT (56).

Stability of MRACS Based on Scheme 2

Stability of MRACS with the scheme 2 is shown as follows. (56) satisfies the next lemma.

Lemma 2 *(Stability of adaptive loop):* The HOT (56) of scheme 2 satisfies the following properties;

P2-1) $\hat{\theta}^{(j)}(\cdot) \in L_\infty$.
P2-2) $J_2(\cdot) \in L_1$.
P2-3) $\dot{\hat{\theta}}(\cdot) \in L_2$.
P2-4) $\hat{\theta}_{n^*}(t) > 0$ for all $t \geq 0$.
P2-5) $\tilde{y}_{\mathrm{N}}(t) \in L_2$ where $\tilde{y}_{\mathrm{N}}(t) := \tilde{y}(t)/N_2(t)$.

proof: The proof can be shown in the same way as one in the lemma 1. Let the positive definite function $V_2(\tilde{\theta}(t))$ be

$$V_2(\tilde{\theta}(t)) := \frac{1}{\gamma_{2n^*}}\left\{-\tilde{\theta}_{n^*}(t) + \theta_{n^*}\ln\frac{\theta_{n^*}}{\hat{\theta}_{n^*}(t)}\right\} + \sum_{i=n^*+1}^{2n}\frac{1}{2\gamma_{2i}}\tilde{\theta}_i^2(t)\,;\ \ \tilde{\theta}_i := \theta_i - \hat{\theta}_i(t) \quad (65)$$

which is defined in the same set as (29). Then, the time differentiation of $V_2(\tilde{\theta}(t)$ evaluated along the trajectory of (56) becomes

$\dot{V}_2(\tilde{\theta}(t)) = -J_2(t)/N_{20}(t) \leq 0$(66)

Therefore, $V_2(\tilde{\theta}(\cdot))$, $\hat{\theta}(\cdot) \in L_\infty$ and $V_2\left(\tilde{\theta}(t)\right)$ converge some constant in [0, $V_2(\tilde{\theta}(0))$]. Hence, P2-1) is proven because $q_2^{(j)}(\cdot)$, $R_2^{(j)}(\cdot) \in L_\infty$ for $j = 0 \sim n^*$. From the integration of (66), P2-2) is proven. P2-3) holds since $\|\,\dot{\hat{\theta}}(t)\,\|^2$ can be evaluated as

$\|\,\dot{\hat{\theta}}(t)\,\|^2 \leq \|\,\Gamma_2(t)\,\|^2\ \{\mathrm{trace}\,R_2(t)\}^2 J_2(t)$(67)

and $J_2(\cdot) \in L_1$. P2-4) is shown by the fact

$\lim_{\hat{\theta}_{n^*}(t)\to 0} n_{\mathrm{c0}}^{\mathrm{T}}\,\Gamma_2(t)\{q_2(t) - R_2(t)\,\hat{\theta}(t)\} = 0$(68)

where $n_{\mathrm{c0}} := [-1, 0^{\mathrm{T}}]^{\mathrm{T}}$ is the normal vector at $\hat{\theta}_{n^*}(t) = 0$ of the convex set C. In addition, P2-5) is proven by P2-2) and the mathematical induction similar to the paper (Tanahashi, 2011). Q. E. D.

From the above lemma 2, the following theorem about the main control loop (closed loop) holds.

Theorem 2 *(Stability):* All variables in the adaptive control system which consists of (1), (49) and (56) are

uniformly bounded in t, and

$\lim_{t\to\infty}\tilde{y}(t) = 0$(69)

proof: In the following, the exponential decaying term is abbreviated. Note that $\varphi(t)$ can be rewritten as

$$\left.\begin{aligned}\varphi(t) &= \textstyle\sum_{i=0}^{n^*-1}\alpha_1(t)(w_i(s)[y])(t)\\ w_i(s) &:= \frac{1}{(s+\lambda)^i}z(s)\end{aligned}\right\}(70)$$

where $z(s)$ is the same transfer vector as (34).

By (52) and procedure similar to the previous section, $\varphi(t)$ can be represented as follows;

$$M(t)\,\varphi(t) = c_{\mathrm{m}}(t) + \left(\bar{z}(s)\left[\tilde{\theta}^{\mathrm{T}}\varphi_{\mathrm{N}}\cdot N_2\right]\right)(t) + \sum_{i=0}^{n^*-1}\alpha_i(t)\left(w_i(s)\left[\tilde{\theta}^{\mathrm{T}}\varphi_{\mathrm{N}}\cdot N_2\right]\right)(t) \quad (71)$$

$$c_{\mathrm{m}}(t) := \sum_{i=0}^{n^*-1}\alpha_i(t)(w_i(s)[y_{\mathrm{m}}])(t) \quad (72)$$

Where $M(t)$ and $\bar{z}(s)$ are defined as (37) and (35) respectively.

Then, $\|\varphi(t)\|^2$ is evaluated as

$\|\varphi(t)\|^2 \le c + c\int_0^t \tilde{y}_{\mathrm{N}}^2(\tau)\,\|\varphi(\tau)\|^2\,\mathrm{d}\tau$(73)

Since $M^{-1}(\cdot) \in L_\infty$, $\alpha_i(\cdot) \in L_\infty$, and P2-5) are satisfied. $c > 0$ is some generic constant. This means

$\|\varphi(t)\|^2 \le c\,\exp\left\{c\int_0^t \tilde{y}_{\mathrm{N}}^2(\tau)\,\mathrm{d}\tau\right\}$(74)

from Bellman-Gronwall lemma (Ioannou, 1996). This means $\varphi(\cdot) \in L_\infty$ from P2-5). Hence, $y(\cdot) \in L_\infty$ from (1). $u(\cdot) \in L_\infty$ is guaranteed from *linear boundedness condition* (Narendra, 1989) or regularity (Sastry, 1989) since the controlled object is a minimum phase system, $y(\cdot) \in L_\infty$ and the fact that $u(t)$ grows at most exponentially from $\hat{\theta}(\cdot) \in L_\infty$.

The result of $u(\cdot), y(\cdot) \in L_\infty$ means $\tilde{y}(\cdot) \in L_\infty \cap L_2$ and $\dot{\varphi}(\cdot) \in L_\infty$. So that, $\tilde{y}(t)$ is uniformly continuous signal from $\hat{\theta}(\cdot), \dot{\hat{\theta}}(\cdot), \varphi(\cdot), \dot{\varphi}(\cdot) \in L_\infty$ and

$\dot{\tilde{y}}(t) = \dot{\hat{\theta}}^{\mathrm{T}}(t)\varphi(t) + \tilde{\theta}^{\mathrm{T}}(t)\dot{\varphi}(t)$(75)

By Barbălat's lemma, these facts lead to zero convergence of $\tilde{y}(t)$. Q. E. D.

Numerical Example

In this section, simple examples will be presented to illustrate the usefulness of the theoretical results of the proposed two schemes.

The followings are settings for the numerical examples. The plant was chosen as

$$y(t) = \left(\frac{1}{s^2 - 0.5s}[u]\right)(t)$$

and the reference model was selected as

$$y_{\mathrm{m}}(t) = \left(\frac{1}{s^2 + 1.4s + 1}[r]\right)(t)$$

where $r(t)$ is the rectangular wave with the period 20[s], varying between 0 and 1. Other settings are as follows;

$\lambda_1 = 1, \lambda_1 = \lambda_2 = 2, \rho_1 = \rho_2 = 0.1,$
$\Gamma_{10} = \Gamma_{20} = 20I_3, \hat{\theta}(0) = [2,\ 0,\ 0]^{\mathrm{T}}$
Then, the plant parameter $\theta = [1,\ 2.5,\ -1.5]^{\mathrm{T}}$.

Fig.1 and Fig.2 are simulation results. These illustrate the usefulness of the theoretical results of the proposed schemes. It can be seen that the tracking error in MRACS with the scheme 2 is small relative to one with the scheme 1.

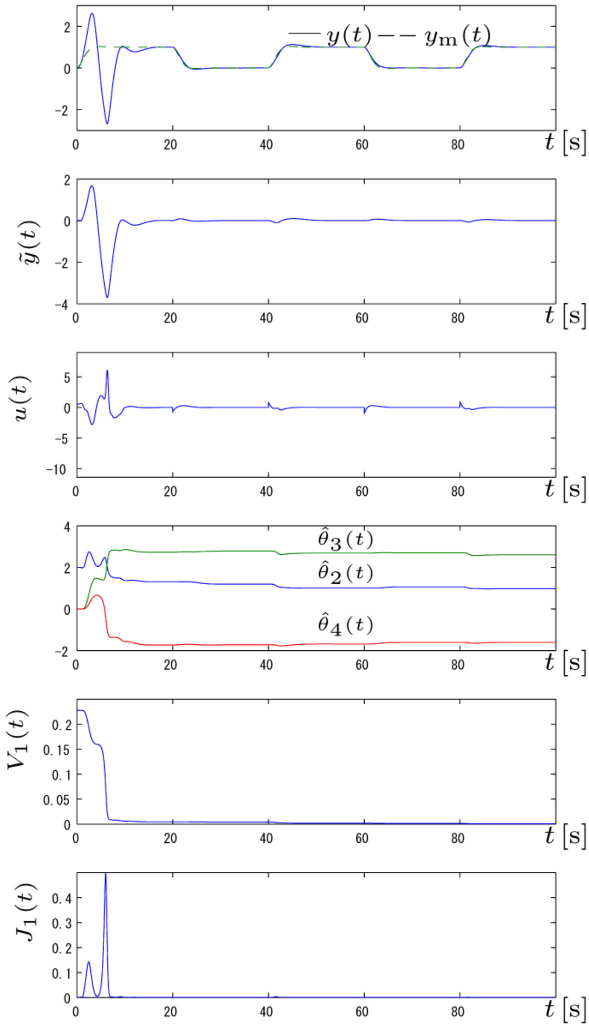

FIG. 1 SIMULATION RESULTS BASED ON SCHEME 1

Conclusions

New two schemes for an indirect MRACS based on SMC are proposed. It is indicated that recursive identifiers and HOTs with smooth projection function for $\hat{\theta}_{n^*}(t) > 0$ in $t \geq 0$ play central roles in both scheme. Extensions to the robust version to unmodeled dynamics and disturbances are the focus of our future studies.

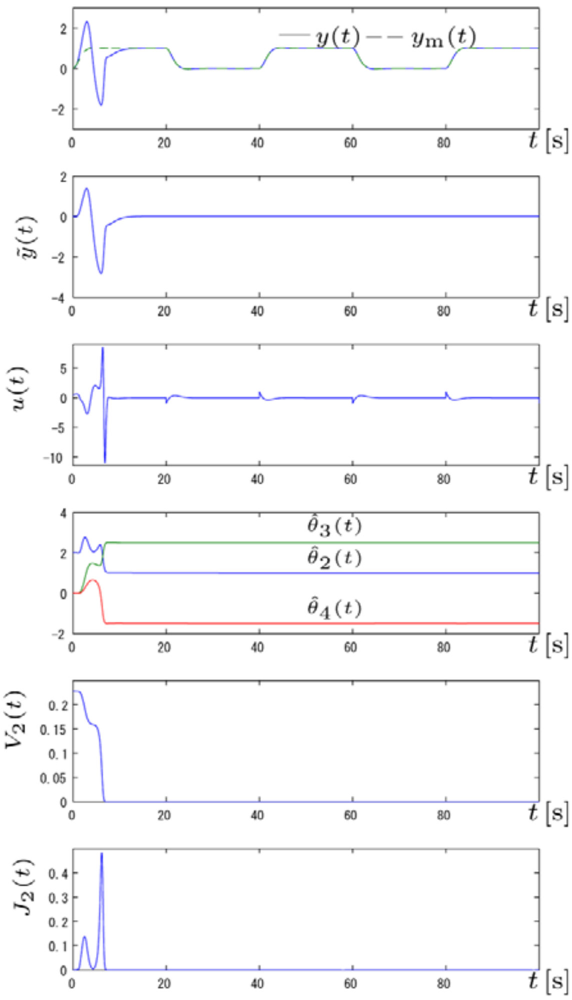

FIG. 2 SIMULATION RESULTS BASED ON SCHEME 2

ACKNOWLEDGMENTS

The authors would like to thank the reviewers who have a rapid peer review.

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